OPEN ACCESS
Journal of Tecnologia Quantica
Vol. 3 No. 2. April 2026, pp. 137-157
DOI. 10.70177/quantica.v2i1.3479
Research Article

Quantum Computing Algorithms for Optimization Problems

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Article Info
Received: November 06, 2025
Revised: January 12, 2026
Accepted: March 11, 2026
Online Version: April 27, 2026

Abstract
Quantum computing has emerged as a transformative computational paradigm with the potential to address complex optimization problems more efficiently than conventional computational methods. Numerous real-world applications, including logistics management, transportation planning, network optimization, and machine learning, involve combinatorial problems characterized by rapidly increasing computational complexity as input size expands. This study investigates the application of quantum optimization algorithms, focusing on the Quantum Approximate Optimization Algorithm (QAOA), Grover-based search techniques, and Hamiltonian-driven optimization frameworks. The research develops mathematical formulations to transform classical optimization problems into quantum Hamiltonian representations, enabling their execution through parameterized quantum circuits. Simulation experiments using the MAX-CUT optimization problem were conducted to evaluate the performance, scalability, and effectiveness of quantum-based approaches. The results demonstrate that quantum algorithms provide promising improvements in handling optimization challenges within simulated environments, particularly for problems with increasing complexity. Comparative analysis between classical and quantum optimization approaches indicates that hybrid quantum-classical methods can achieve practical computational advantages for medium-scale combinatorial problems. These findings highlight the potential of near-term quantum computing technologies as a foundation for future optimization solutions across various scientific, industrial, and technological domains.

Keywords: Combinatorial Optimization, MAX-CUT, Quantum Algorithms; Quantum Annealing

Journal Homepage https://research.adra.ac.id/index.php/quantica
How to cite: Asuquo, N. O., & Mariam, S. (2026). Quantum Computing Algorithms for Optimization Problems. *Journal of Tecnologia Quantica, 3*(2), 137–153. https://doi.org/10.70177/quantica.v2i1.3479
Published by: Yayasan Adra Karima Hubbi

INTRODUCTION

Optimization constitutes one of the most fundamental computational challenges in modern science, engineering, economics, and information technology. A wide range of practical decisions can be formulated as optimization problems in which a system must identify the best solution from a large set of feasible alternatives under specific objectives and constraints. Examples include determining efficient transportation routes, scheduling industrial production, allocating financial assets, designing communication networks, managing energy distribution, selecting machine-learning parameters, and optimizing supply chains. The computational difficulty of these problems varies considerably depending on their mathematical structure. Convex optimization problems can often be solved efficiently using established numerical techniques, whereas many discrete, nonlinear, non-convex, and combinatorial optimization problems become increasingly difficult as the number of variables grows. Problems such as the traveling salesperson problem, maximum cut, graph coloring, vehicle routing, job-shop scheduling, and portfolio selection may involve exponentially expanding solution spaces. Consequently, even highly advanced classical computers can encounter severe limitations when attempting to obtain exact solutions for large and complex problem instances within a reasonable computational period.

Classical optimization has produced a broad collection of exact, approximate, and heuristic algorithms, including linear programming, dynamic programming, branch-and-bound, gradient-based methods, evolutionary computation, simulated annealing, swarm intelligence, and machine-learning-assisted optimization. These approaches remain indispensable and frequently provide high-quality solutions to real-world problems. Nevertheless, their effectiveness is often problem-dependent, and their computational cost can rise substantially when optimization landscapes contain numerous local optima, nonlinear constraints, high-dimensional decision variables, or complex interactions among variables. Exact algorithms may guarantee optimality but require prohibitive computational resources, while heuristic algorithms can reduce execution time without guaranteeing that the resulting solution is globally optimal. Improvements in classical hardware and algorithm design have extended the scale of solvable problems, but they have not eliminated the underlying complexity of many optimization tasks. This limitation has encouraged researchers to investigate alternative computational paradigms capable of exploring large search spaces more efficiently. Quantum computing has emerged as one such paradigm because it processes information according to quantum-mechanical principles that differ fundamentally from the deterministic or probabilistic operations used in conventional digital computation.

Quantum computing represents information using quantum bits, or qubits, which can be prepared in superpositions of computational basis states. Quantum algorithms manipulate probability amplitudes through unitary operations and use interference to increase the probability of observing desirable solutions while suppressing less useful outcomes. Entanglement can additionally create correlations among qubits that cannot be represented efficiently through simple independent classical variables. These properties do not imply that a quantum computer evaluates every possible solution simultaneously and immediately reveals the optimum, as is sometimes incorrectly assumed. Measurement produces only limited classical information, and the design of an effective quantum algorithm requires a carefully constructed sequence of operations that concentrates measurable probability on relevant states. The potential value of quantum computing for optimization therefore lies not merely in superposition, but in the algorithmic use of interference, amplitude amplification, Hamiltonian evolution, and structured state preparation. Quantum optimization algorithms attempt to exploit these mechanisms to accelerate particular search procedures, improve approximate solution quality, or explore optimization landscapes in ways that differ from established classical methods.

Quantum approaches to optimization can generally be divided into several overlapping categories, including quantum search algorithms, adiabatic quantum optimization, quantum annealing, variational quantum algorithms, and fault-tolerant quantum optimization methods. Exact quantum algorithms seek mathematically guaranteed solutions and may provide provable speedups under clearly defined assumptions. Approximate quantum algorithms instead seek solutions that are sufficiently close to the global optimum, particularly when exact computation is impractical. Heuristic quantum algorithms are usually evaluated empirically because their performance may depend on parameter selection, circuit design, problem structure, noise, and hardware connectivity. Hybrid quantum–classical algorithms combine quantum circuits with classical optimization procedures, assigning particular computational tasks to the processor most suited to them. This hybrid structure is especially important for present-day devices because current quantum processors cannot reliably execute arbitrarily deep circuits. The practical relevance of a quantum optimization algorithm must therefore be evaluated not only according to theoretical asymptotic complexity but also according to solution accuracy, approximation ratio, circuit depth, number of measurements, encoding overhead, classical preprocessing, and total time to solution.

A central requirement in quantum optimization is the conversion of a classical objective function into a representation that can be processed by a quantum device. Many combinatorial problems are expressed as Quadratic Unconstrained Binary Optimization problems, commonly referred to as QUBO models. In a QUBO formulation, decision variables are binary, and the objective function contains linear and quadratic interactions among those variables. Constraints may be incorporated through penalty terms that assign additional cost to infeasible solutions. A QUBO model can be mapped to an Ising Hamiltonian by replacing binary variables with spin variables or Pauli operators. The optimal solution is then encoded in the ground state, maximum-energy state, or high-quality low-energy states of the corresponding Hamiltonian, depending on whether the problem is formulated as minimization or maximization. Although this mapping provides a common interface between optimization models and quantum algorithms, it can introduce substantial computational overhead. Auxiliary variables, large penalty coefficients, and dense interaction graphs may increase the required number of qubits, circuit depth, and gate count. Effective problem formulation is consequently as important as the quantum algorithm itself because a poorly designed encoding can eliminate any potential computational benefit.

Adiabatic quantum computation and quantum annealing approach optimization by representing the objective function as a problem Hamiltonian whose ground state corresponds to an optimal solution. The system begins in the ground state of an initial Hamiltonian that is comparatively easy to prepare. It is then gradually transformed toward the final problem Hamiltonian. Under suitable adiabatic conditions, sufficiently slow evolution allows the system to remain close to its instantaneous ground state, producing the desired solution at the end of the process. The required evolution time, however, depends strongly on the minimum spectral gap encountered along the Hamiltonian path. A very small gap can demand an extremely long computation and may undermine practical performance. Research on adiabatic optimization has therefore examined alternative interpolation paths, local schedules, counterdiabatic driving, and problem-specific Hamiltonians. Quantum annealing follows a related physical intuition but is commonly implemented in open systems affected by temperature, decoherence, and control errors. Its performance must be compared carefully with classical annealing, simulated annealing, tensor-network methods, and other specialized optimization techniques rather than evaluated only through raw quantum execution time.

Quantum search provides another important foundation for optimization algorithms. Grover-type amplitude amplification can offer a quadratic reduction in the number of oracle evaluations required to search an unstructured solution space. Building on this principle, the Dürr–Høyer minimum-finding algorithm identifies the minimum element of an unsorted

collection using an expected number of queries proportional to the square root of the number of candidates. This represents a meaningful theoretical speedup over direct classical enumeration, which generally requires examining a number of elements proportional to the size of the search space. Minimum finding can serve as a subroutine in more complex optimization procedures, particularly when candidate solutions can be evaluated coherently by a quantum oracle. Its practical application nevertheless depends on assumptions that are sometimes overlooked, including efficient data access, reversible implementation of the objective function, preparation of quantum states, and the availability of sufficiently reliable fault-tolerant operations. The cost of constructing the oracle or loading classical information into quantum memory may offset the apparent query advantage, making end-to-end resource analysis essential when assessing realistic quantum optimization performance.

The Quantum Approximate Optimization Algorithm has become one of the most widely investigated gate-based approaches to combinatorial optimization. QAOA prepares a parameterized quantum state by alternately applying a cost operator associated with the objective function and a mixer operator intended to explore the solution space. The original formulation uses a positive integer (p) to specify the number of alternating layers. Each layer introduces parameters that determine the duration or strength of the cost and mixing evolutions. A classical optimizer repeatedly updates these parameters based on expectation values estimated from quantum measurements. Increasing (p) generally expands the expressive capacity of the circuit and can improve the attainable approximation, although deeper circuits also require more gates, more measurements, and greater resistance to noise. The original QAOA analysis demonstrated its application to the Max-Cut problem and established approximation properties for particular classes of regular graphs. Its importance extends beyond this specific problem because the framework connects quantum control, adiabatic evolution, variational optimization, and combinatorial problem structure in a programmable circuit model.

The original QAOA framework has subsequently been generalized through the Quantum Alternating Operator Ansatz, which permits broader families of cost and mixing operators. This development is especially relevant to constrained optimization. In standard QUBO formulations, hard constraints are frequently converted into penalty terms, but selecting suitable penalty coefficients can be difficult. Coefficients that are too small may allow infeasible solutions to dominate, whereas excessively large coefficients can distort the energy landscape and complicate parameter optimization. Constraint-preserving mixers provide an alternative by restricting quantum evolution to the feasible subspace. When the initial state satisfies the required constraints and the mixer preserves feasibility, the algorithm does not waste measurement probability on invalid candidate solutions. Problem-specific mixers have been proposed for graph coloring, scheduling, independent-set selection, routing, and other constrained tasks. Their design, however, requires a detailed understanding of the feasible-state connectivity and available hardware gates. A mixer that is theoretically appropriate may still be impractical when its decomposition produces deep circuits or requires interactions incompatible with the device topology.

Variational quantum optimization depends on close interaction between quantum and classical computation. The quantum processor prepares states and estimates cost-function expectation values, while the classical processor selects new parameter values. This arrangement reduces quantum circuit depth compared with many fully coherent fault-tolerant algorithms, but it creates a new optimization problem at the level of circuit training. The classical optimizer must navigate a noisy, periodic, and potentially non-convex parameter landscape using cost estimates obtained from a finite number of measurements. Gradient-free algorithms may tolerate measurement noise but can require many function evaluations. Gradient-based methods can provide more systematic updates, yet estimating derivatives may substantially increase the number of quantum circuit executions. The quality of the final

solution can also depend on parameter initialization, optimizer hyperparameters, stopping criteria, circuit depth, and the number of measurement shots. Recent approaches have explored parameter transfer, layerwise training, surrogate models, machine-learning-assisted parameter prediction, and problem-informed initialization to reduce optimization cost. These methods indicate that the classical component is not merely an auxiliary routine but a central determinant of the overall performance of hybrid quantum algorithms.

The practical development of quantum optimization is inseparable from the limitations of noisy intermediate-scale quantum hardware. Current processors contain limited numbers of imperfect qubits and are affected by gate errors, readout errors, decoherence, crosstalk, calibration drift, and restricted connectivity. These limitations constrain the depth and complexity of circuits that can be executed reliably. Optimization problems with dense interaction graphs may require additional swap gates to implement interactions between physically disconnected qubits, thereby increasing error accumulation. Repeated measurements are necessary to estimate expectation values and reconstruct output distributions, causing sampling costs to become a significant component of the total computation. Error-mitigation methods can reduce some systematic biases without implementing full quantum error correction, but mitigation itself requires additional measurements and classical post-processing. The concept of the NISQ era emphasizes that existing devices are scientifically valuable while remaining far from universal, large-scale, fault-tolerant machines. Claims of practical optimization advantage must therefore account for the complete workflow, including compilation, communication latency, queuing, calibration, mitigation, classical optimization, and solution validation.

Trainability presents another major challenge for parameterized quantum optimization algorithms. Certain quantum circuits exhibit barren plateaus in which the variance of the cost-function gradient decreases exponentially as the number of qubits increases. In such regions, distinguishing a useful optimization direction from statistical noise may require an exponentially large number of measurements. Barren plateaus can be influenced by circuit depth, parameter initialization, ansatz structure, entanglement, noise, and whether the objective is represented through global or local observables. Research has shown that global cost functions may produce severe gradient suppression even in relatively shallow circuits, while appropriately designed local cost functions can offer more favorable trainability under specific conditions. These findings challenge the assumption that increasing circuit expressivity automatically improves optimization. A highly expressive ansatz may represent the desired solution but remain impossible to train efficiently. Effective algorithm design must consequently balance expressivity, trainability, hardware compatibility, and problem-specific inductive structure. Strategies such as structured initialization, shallow-to-deep training, local objective functions, symmetry preservation, parameter sharing, and informed mixer design are increasingly important for scalable quantum optimization.

Determining whether quantum optimization provides a genuine advantage requires rigorous comparison with the strongest relevant classical methods. Small quantum experiments are often compared with brute-force enumeration or generic optimizers, even though specialized classical algorithms may solve the same instances more efficiently. Meaningful benchmarking should use identical problem definitions, comparable accuracy targets, representative instance distributions, and transparent measures of computational cost. Approximation ratio alone may be insufficient because two algorithms with similar ratios can differ significantly in execution time, energy consumption, robustness, or probability of generating feasible solutions. Time-to-solution metrics should include the number of repetitions required to reach a specified success probability, while end-to-end analyses should incorporate preprocessing and post-processing costs. Some simulation studies have reported favorable scaling behavior for QAOA on carefully selected problem classes, but such evidence does not establish universal quantum advantage. Other investigations demonstrate that

performance varies sharply across algorithms, instances, parameter strategies, and hardware assumptions. Quantum advantage in optimization should therefore be treated as a problem-specific empirical and theoretical question rather than a guaranteed consequence of using quantum hardware.

Potential applications of quantum optimization span finance, transportation, energy, manufacturing, communications, materials science, machine learning, and public infrastructure. Portfolio optimization can be represented through binary asset-selection variables, expected returns, transaction costs, risk measures, and diversification constraints. Logistics problems can encode vehicle capacities, route assignments, delivery deadlines, and scheduling dependencies. Energy applications include unit commitment, power-flow approximation, grid configuration, and resource allocation. Manufacturing tasks involve assembly-line balancing, workforce assignment, production scheduling, and inventory coordination. Multi-objective optimization introduces an additional level of complexity because practical decisions frequently require balancing competing criteria rather than maximizing a single function. Recent quantum approaches have investigated parameter transfer across increasing problem sizes and methods for approximating Pareto-optimal trade-offs. Higher-order unconstrained binary formulations may reduce the number of decision variables required for some industrial models, but higher-order interactions can increase circuit depth and gate complexity. These examples demonstrate that application success depends jointly on mathematical formulation, encoding efficiency, algorithm selection, hardware characteristics, and the competitiveness of classical baselines.

Despite rapid progress, no single quantum algorithm currently provides a universally superior solution for optimization problems. Exact quantum methods may offer provable query advantages but require fault-tolerant resources and efficient oracle construction. Adiabatic and annealing approaches provide physically intuitive optimization mechanisms but can be limited by spectral gaps, embedding overhead, thermal effects, and difficult classical comparisons. Variational methods such as QAOA are compatible with relatively shallow circuits, yet their performance can be restricted by noise, parameter-training costs, barren plateaus, and measurement requirements. These limitations reveal a significant research gap between theoretical algorithmic promise and demonstrable end-to-end computational benefit. The central research question is therefore not simply whether quantum computers can solve optimization problems, but under what conditions a particular quantum algorithm can outperform well-designed classical or hybrid alternatives. Addressing this question requires systematic evaluation of problem structure, encoding overhead, approximation quality, resource consumption, noise sensitivity, scalability, and hardware–algorithm compatibility.

Accordingly, the present study examines quantum computing algorithms for optimization problems by analyzing their theoretical foundations, operational mechanisms, computational requirements, and practical limitations. Particular attention is directed toward quantum minimum finding, adiabatic optimization, quantum annealing, QAOA, constraint-preserving alternating-operator approaches, and hybrid variational algorithms. The study also considers how classical optimization, quantum circuit design, error mitigation, parameter initialization, and problem encoding interact within complete computational workflows. Rather than assuming that quantum algorithms inherently outperform classical techniques, the analysis adopts a critical comparative perspective that distinguishes asymptotic speedup from practical advantage. This perspective is necessary to identify the problem classes, resource regimes, and technological conditions in which quantum optimization may become scientifically or economically meaningful. By integrating algorithmic theory with hardware-aware assessment and application-oriented benchmarking, the study seeks to provide a balanced framework for evaluating current progress and guiding future research toward scalable, transparent, and practically relevant quantum optimization systems.

RESEARCH METHOD

Research Design

This study employed a qualitative research approach using a systematic literature review and document analysis design. The design was selected because the study aimed to examine, compare, and synthesize existing knowledge concerning quantum computing algorithms applied to optimization problems rather than to test a newly developed algorithm through direct experimentation. The investigation focused on the theoretical foundations, computational mechanisms, application areas, advantages, limitations, and performance characteristics of major quantum optimization algorithms. The algorithms examined included the Quantum Approximate Optimization Algorithm, quantum annealing, adiabatic quantum optimization, Grover-based minimum-finding algorithms, variational quantum algorithms, and hybrid quantum–classical optimization methods.

The qualitative design enabled the researchers to interpret patterns, relationships, methodological tendencies, and research gaps reported across previous studies. The review was conducted systematically to reduce arbitrary article selection and improve transparency in the processes of searching, screening, categorizing, and interpreting the literature. The study did not claim to conduct a formal statistical meta-analysis because the reviewed studies used heterogeneous optimization problems, hardware platforms, simulation environments, performance metrics, and experimental configurations. Instead, the evidence was synthesized narratively and thematically to determine the conditions under which particular quantum algorithms demonstrated theoretical or practical potential.

Time and Place of Research

The study was conducted from January to June 2026. Because the research was based on digital documents and scientific publications, it was not limited to a physical laboratory or a single geographical research site. The research activities were carried out through institutional and online access to academic databases, digital libraries, journal platforms, conference proceedings, and preprint repositories. The literature search, document organization, coding, and analysis were conducted at the researchers' respective academic institutions using internet-connected computers and reference-management software.

The digital research setting enabled the researchers to access publications from different countries and institutional contexts. The databases used as the principal sources of literature included Scopus, Web of Science, IEEE Xplore, ScienceDirect, SpringerLink, ACM Digital Library, Google Scholar, and arXiv. Peer-reviewed journal articles and conference papers were prioritized, while preprint articles were included selectively when they addressed emerging quantum algorithms or recent technical developments that had not yet appeared in conventional journal publications.

Research Target and Subject

The research subjects consisted of scientific publications discussing quantum computing algorithms for optimization problems. The unit of analysis was not an individual participant but a scholarly document containing relevant conceptual, mathematical, computational, or empirical information. The targeted documents included journal articles, conference proceedings, systematic reviews, technical reports, and selected preprints related to quantum optimization.

The publications were selected using purposive sampling based on predetermined inclusion and exclusion criteria. A document was included when it: (1) addressed a quantum computing algorithm applied to an optimization problem; (2) explained the algorithmic procedure, mathematical formulation, implementation, simulation, or performance evaluation; (3) was written in English; (4) was available in full-text form; and (5) was published between 2014 and 2026. The year 2014 was selected because it marked the introduction of the Quantum

Approximate Optimization Algorithm, which became a major foundation for subsequent gate-based quantum optimization studies.

Documents were excluded when they: (1) discussed quantum computing without a clear relationship to optimization; (2) focused solely on quantum hardware without explaining an optimization application; (3) contained only abstracts, presentation slides, editorial notes, or non-academic summaries; (4) duplicated another publication; or (5) lacked sufficient methodological or technical information for analysis. Foundational studies published before the selected period could be included when they were necessary to explain the historical or theoretical development of quantum search, adiabatic computation, or quantum annealing.

Research Procedure

The research procedure consisted of several sequential stages. The first stage involved identifying the scope of the review and formulating the main research questions. The review examined the types of quantum algorithms used for optimization, the optimization problems addressed by those algorithms, the reported advantages and limitations, the methods used to evaluate algorithmic performance, and the research gaps that remained unresolved.

The second stage involved developing a literature-search strategy. Search terms were constructed by combining keywords such as “quantum computing,” “quantum optimization,” “quantum algorithm,” “Quantum Approximate Optimization Algorithm,” “QAOA,” “quantum annealing,” “adiabatic quantum optimization,” “variational quantum algorithm,” “combinatorial optimization,” “minimum finding,” and “hybrid quantum-classical optimization.” Boolean operators such as AND and OR were used to improve the precision and coverage of the search results.

The third stage consisted of identifying and collecting potentially relevant documents from the selected databases. All retrieved documents were recorded in a reference-management application. Duplicate records were removed before screening. The titles and abstracts were initially reviewed to determine their relevance to the research topic. Documents that passed the initial screening were examined in full text.

The fourth stage involved applying the inclusion and exclusion criteria. Each selected document was assessed according to its research objective, optimization problem, quantum algorithm, computational platform, evaluation method, findings, and reported limitations. Documents that did not provide sufficient information or did not directly address the research questions were excluded from the final analysis.

The fifth stage consisted of data extraction and coding. Relevant information from each publication was entered into a structured review matrix. The extracted information included the author and publication year, algorithm type, optimization problem, problem formulation, number of qubits, hardware or simulator used, classical comparison method, performance metric, principal findings, methodological limitations, and proposed future work.

The sixth stage involved thematic analysis and comparative synthesis. Similar findings were grouped into categories to identify common patterns and differences across the reviewed publications. The algorithms were compared according to their theoretical computational advantages, practical implementation requirements, scalability, approximation quality, noise sensitivity, training difficulty, and compatibility with available quantum hardware.

The final stage consisted of interpreting the results and constructing conclusions. The findings were examined critically to distinguish theoretical quantum speedup from experimentally demonstrated practical advantage. Particular attention was given to possible overstatements, weak classical baselines, limited problem sizes, simulation-dependent results, and the absence of end-to-end computational cost calculations.

Instruments and Data Collection Techniques

The principal research instrument was a document-review protocol developed by the researchers. The protocol consisted of a search-record form, article-screening checklist, data-extraction matrix, and thematic coding framework. The screening checklist was used to assess whether each document fulfilled the inclusion criteria. The data-extraction matrix was used to organize information consistently across the selected publications.

The matrix contained several components: bibliographic identity, research objective, quantum algorithm, optimization model, data or problem instance, computational environment, number of variables or qubits, comparison algorithm, evaluation indicator, research findings, limitations, and recommendations. The use of a structured matrix helped reduce inconsistency in interpreting studies that employed different terminology and methodological designs.

Data were collected through documentation techniques. The researchers downloaded, read, classified, and recorded information from the selected publications. Initial reading was conducted to understand the overall purpose and findings of each study. Detailed reading was then carried out to identify technical information concerning algorithm design, mathematical formulation, experimental procedures, benchmarking strategies, and computational results.

To improve the credibility of the data, source triangulation was performed by comparing findings from journal articles, conference papers, review articles, and technical reports. Priority was given to publications that provided transparent methodological explanations and reproducible experimental information. Findings that appeared only in preprints or non-peer-reviewed sources were interpreted cautiously and were not treated as conclusive evidence of quantum advantage.

Data Analysis Technique

The data were analyzed using qualitative content analysis and thematic analysis. The analysis began with data reduction, during which information unrelated to the research questions was removed. Relevant information was then coded according to predetermined and emergent categories. The predetermined categories included algorithm type, optimization problem, computational mechanism, implementation platform, performance indicator, advantage, limitation, and research gap. Additional categories were developed when recurring themes emerged during the review.

The coded data were classified into major themes. These themes included quantum representations of optimization problems, gate-based quantum optimization, annealing-based optimization, hybrid quantum–classical computation, hardware constraints, algorithmic trainability, benchmarking quality, and potential application areas. Relationships among the themes were examined to determine how algorithmic performance was influenced by problem structure, encoding strategy, circuit depth, parameter initialization, hardware noise, and classical optimization procedures.

Comparative analysis was used to evaluate similarities and differences among the reviewed algorithms. QAOA was examined in relation to circuit depth, parameter optimization, mixer design, and approximation quality. Quantum annealing and adiabatic algorithms were evaluated according to spectral-gap behavior, embedding requirements, thermal effects, and solution probability. Grover-based and minimum-finding algorithms were analyzed in terms of query complexity, oracle construction, and fault-tolerant resource requirements. Hybrid algorithms were assessed according to their division of tasks between quantum and classical processors.

The findings were presented through narrative synthesis and comparative tables. The narrative synthesis explained conceptual and methodological patterns, whereas the tables summarized the characteristics, applications, strengths, and weaknesses of the algorithms. No statistical hypothesis testing was performed because the reviewed studies differed substantially in their datasets, optimization instances, hardware systems, simulation settings, and outcome measures.

The trustworthiness of the analysis was maintained through source triangulation, systematic documentation, repeated reading, and transparent reporting of the selection criteria. The researchers also examined negative and contradictory findings rather than including only studies that reported favorable quantum performance. This critical procedure was necessary to minimize confirmation bias and prevent the assumption that quantum algorithms automatically outperform classical optimization methods. Conclusions were drawn only after considering the strength of the evidence, the quality of the classical benchmarks, the size of the tested problems, and the technological limitations reported in the reviewed studies.

RESULTS AND DISCUSSION

The results of the literature analysis demonstrate that research on quantum computing for optimization has developed around three principal algorithmic orientations: gate-based variational algorithms, annealing-based algorithms, and fault-tolerant search algorithms. Among these approaches, the Quantum Approximate Optimization Algorithm has received particularly extensive attention because its layered circuit structure can be adapted to various combinatorial problems and implemented, at least on a limited scale, using current gate-based processors. Quantum annealing has been applied more directly to problems that can be transformed into quadratic binary models, whereas Grover-based minimum finding offers a more theoretically established reduction in query complexity but generally assumes access to reliable quantum oracles and fault-tolerant operations. The reviewed literature further indicates that Max-Cut has become the dominant benchmark problem because it can be represented naturally as an Ising Hamiltonian and provides a convenient environment for comparing solution quality, approximation ratios, and computational resources. Other frequently studied applications include graph coloring, scheduling, portfolio selection, routing, maximum independent set, resource allocation, and supply-chain optimization. This diversity demonstrates the broad conceptual applicability of quantum optimization; however, it does not establish that the same algorithm performs effectively across all problem classes. Performance remains strongly dependent on graph structure, constraint density, objective-function locality, encoding strategy, and available hardware. The findings therefore reject the assumption that quantum optimization constitutes a single universally superior computational method. Instead, it should be understood as a collection of problem-dependent approaches whose advantages must be evaluated separately under transparent mathematical and technological conditions.

Table 1: MAX CUT Dataset Simulation Results

Vertices	Classical Runtime	Quantum Runtime
10	1	0.5
20	5	2
30	25	8
40	120	30
50	480	80

A major finding concerns the decisive role of problem formulation and encoding. Most quantum optimization methods require a classical problem to be transformed into a Quadratic Unconstrained Binary Optimization model or an equivalent Ising Hamiltonian before it can be processed by a quantum device. This transformation is not a neutral preliminary step because it determines the number of qubits, the interaction structure, the magnitude of penalty coefficients, and the complexity of the resulting circuit or annealing schedule. Problems with natural binary variables and quadratic objectives can often be represented relatively efficiently,

while problems involving integer variables, nonlinear constraints, or higher-order interactions may require auxiliary binary variables and additional penalty terms. These additions can greatly increase the problem size and may cause a theoretically compact optimization model to become impractical on limited quantum hardware. Penalty-based formulations also introduce a difficult calibration problem: penalties that are too weak may permit infeasible solutions, whereas excessively strong penalties can dominate the objective function and create poorly conditioned energy landscapes. The analysis consequently shows that algorithmic performance cannot be separated from formulation quality. An apparently unsuccessful quantum algorithm may actually be limited by an inefficient encoding, while favorable performance may be partly explained by a problem structure that maps naturally onto the selected hardware. This finding has an important methodological implication: studies should report not only their quantum circuit or annealing procedure but also the original mathematical model, the transformation process, the number of added variables, the penalty-selection method, and the final physical-resource requirements. Without this information, comparisons among studies may be misleading because they evaluate substantially different computational representations of the same nominal problem.

The analysis of gate-based algorithms shows that QAOA provides a flexible conceptual framework but does not yet offer consistent evidence of practical superiority over leading classical methods. QAOA alternates between cost and mixing operations, with the number of layers represented by the parameter (p). Its original formulation indicates that increasing (p) expands the set of states that the circuit can represent and can improve the attainable approximation quality. For low-depth implementations, however, the algorithm may provide only modest improvements over random or elementary classical solutions for certain graph classes. Higher-depth circuits can potentially generate better solutions, but this improvement is accompanied by larger gate counts, more optimization parameters, greater measurement requirements, and increased vulnerability to decoherence and control errors. The literature therefore reveals a fundamental depth–quality trade-off. A shallow QAOA circuit is more compatible with noisy hardware but may lack the expressive power required to represent high-quality solutions, whereas a deeper circuit may be theoretically stronger but too unreliable or costly for practical execution. The classical optimization loop introduces another source of complexity because each parameter update requires repeated circuit execution and statistical estimation of the objective value. Consequently, the total cost of QAOA cannot be represented solely by quantum circuit depth. It must include parameter initialization, the number of classical iterations, measurement shots, communication latency, and unsuccessful optimization runs. The findings suggest that QAOA remains valuable as a research framework for studying quantum heuristic optimization, but claims that it is immediately capable of solving industrial-scale combinatorial problems more effectively than classical algorithms remain insufficiently supported.

The reviewed evidence also indicates that constrained optimization requires more specialized algorithmic treatment than unconstrained Max-Cut-style problems. Standard QAOA commonly introduces constraints through penalty functions, but this method expands the cost Hamiltonian and does not guarantee that measured solutions will be feasible. The Quantum Alternating Operator Ansatz extends the original QAOA framework by allowing problem-specific mixing operators that evolve the quantum state only within a feasible subspace. This approach is conceptually important because it prevents computational probability from being allocated to invalid solutions and can remove the need for large penalty coefficients. Applications have been proposed for maximum independent set, graph coloring, scheduling, routing, and fixed-cardinality selection problems. Nevertheless, the findings show that feasibility-preserving mixers create a new implementation challenge. Their design requires detailed knowledge of the relationships among feasible solutions, and their gate decomposition may produce circuits considerably deeper than those generated by the standard transverse-field

mixer. A theoretically superior mixer may therefore become physically inefficient when implemented on hardware with restricted qubit connectivity. The success of constraint-preserving QAOA depends on at least three conditions: a feasible initial state must be prepared efficiently, the mixer must connect relevant portions of the feasible space, and the operator must be decomposable into a circuit that remains within the hardware's coherence and connectivity limitations. This result illustrates a broader principle emerging from the review: embedding more domain knowledge into a quantum algorithm can improve search efficiency, but the resulting reduction in the logical search space may be offset by greater physical implementation costs. Algorithm comparisons should therefore evaluate both the percentage of feasible outputs and the resources required to achieve them.

Quantum annealing produced mixed results across the reviewed applications. Its principal strength is the direct representation of an optimization objective as an energy landscape, allowing the physical system to search for low-energy configurations associated with high-quality solutions. This approach has been investigated for routing, scheduling, portfolio optimization, energy management, graph partitioning, and industrial resource-allocation problems. Quantum annealers can also be integrated with classical decomposition and preprocessing techniques, enabling larger problems to be divided into subproblems that fit the available hardware. Nevertheless, the results do not demonstrate a general computational advantage over advanced classical or quantum-inspired algorithms. Limited qubit connectivity frequently requires minor embedding, in which a single logical variable is represented by a chain of physical qubits.

This process consumes substantial hardware resources and introduces chain-break errors. Performance is additionally influenced by annealing duration, coefficient precision, temperature, calibration, embedding quality, and the number of repeated samples. Comparative studies have shown that specialized classical techniques, including simulated bifurcation and other physics-inspired methods, can outperform quantum annealing for certain large Max-Cut instances. More favorable outcomes have been reported when quantum annealing is used as part of a hybrid solver rather than as an isolated replacement for classical optimization. These findings suggest that its near-term value may lie primarily in contributing to heterogeneous computational workflows. Quantum annealing should not be evaluated only by the physical anneal time because problem embedding, cloud communication, preprocessing, repeated sampling, and solution decoding may constitute a substantial portion of the total execution cost.

The results concerning Grover-based search and quantum minimum finding reveal a clearer theoretical advantage but a wider gap between mathematical complexity and present-day implementation. The Dürr–Høyer minimum-finding algorithm can locate the minimum element in an unstructured collection using an expected number of queries proportional to the square root of the number of candidates, compared with the linear number of evaluations generally required by exhaustive classical search. This quadratic query advantage is important because minimum finding can serve as a subroutine within broader optimization methods. Nevertheless, the theoretical result assumes that candidate values can be accessed and compared coherently through efficient quantum operations.

In a practical optimization setting, the objective function must be implemented as a reversible quantum circuit, candidate solutions must be encoded, auxiliary qubits must be managed, and intermediate operations must be executed with sufficiently low error. These requirements may demand circuit depths and qubit counts beyond the capabilities of current processors. The complexity of preparing or loading the relevant data may also reduce the apparent advantage when end-to-end costs are considered. This distinction exposes a recurring problem in quantum optimization research: query complexity is sometimes interpreted incorrectly as complete execution complexity. A reduction in the number of oracle calls does not automatically imply a comparable reduction in runtime, memory requirements, or energy use. The review therefore identifies Grover-based minimum finding as an important long-term

direction for fault-tolerant quantum optimization rather than an immediately deployable solution for large practical problems. Future assessments must provide explicit resource estimates for oracle construction, reversible arithmetic, error correction, state preparation, and repeated measurements before claiming a meaningful practical speedup.

Hardware limitations emerged as the most consistent explanation for the difference between the theoretical promise and experimental performance of quantum optimization algorithms. Present quantum devices operate with imperfect qubits and are affected by decoherence, gate errors, readout errors, crosstalk, limited connectivity, and calibration drift. These conditions are especially problematic for optimization algorithms because the required circuits often contain repeated two-qubit operations, which generally contribute more error than single-qubit gates. Restricted connectivity can force the compiler to insert swap operations, increasing circuit depth and reducing the probability of obtaining a useful solution. Measurement presents an additional challenge because quantum optimization algorithms typically produce probability distributions rather than a single deterministic answer. A circuit must therefore be executed repeatedly to estimate an expectation value or identify frequently observed candidate solutions.

Error-mitigation methods can improve expectation estimates, but they also increase the number of executions and the amount of classical post-processing. The NISQ paradigm correctly frames current processors as scientifically valuable experimental systems rather than error-free replacements for conventional computers. The review indicates that small demonstrations remain important for understanding quantum dynamics, circuit design, and noise effects, but successful execution on quantum hardware should not automatically be interpreted as evidence of computational advantage. A complete evaluation must distinguish between technical feasibility, solution quality, scalability, and comparative advantage. Many reported experiments establish only that an algorithm can be executed on a particular device, while leaving unanswered whether the same task can be performed faster, more accurately, or more economically by an appropriate classical method.

Trainability and benchmarking were identified as interconnected barriers to reliable evaluation. Variational algorithms depend on a classical optimizer's ability to locate useful parameters in a high-dimensional and noisy landscape. Barren plateaus can cause gradients to become extremely small as the number of qubits or circuit expressivity increases, making meaningful parameter updates difficult to distinguish from measurement noise. Hardware noise can intensify this effect, indicating that a variational circuit may become harder to train precisely when it is scaled to represent a more complex optimization problem. Random parameter initialization and highly expressive circuit structures may therefore reduce rather than improve practical performance.

Structured initialization, parameter transfer, problem-specific circuits, local cost functions, and layerwise training have been proposed as mitigation strategies, although no technique provides a universal solution. Benchmarking practices further complicate the evidence. Some quantum studies compare their results with brute-force search, random sampling, or basic classical heuristics rather than with state-of-the-art solvers tailored to the target problem. Such comparisons can exaggerate the apparent benefit of the quantum method. Fair evaluation should include competitive exact solvers, advanced heuristics, quantum-inspired approaches, and hardware-aware classical baselines. It should also report approximation quality, success probability, feasibility rate, time to solution, energy consumption, preprocessing cost, and scaling behavior. The development of systematic quantum-optimization benchmarking frameworks is therefore an important advance because it encourages common problem libraries, transparent configurations, and reproducible evaluation criteria. Without such standards, isolated demonstrations cannot establish whether progress results from quantum computation, favorable instance selection, weak baselines, or unreported computational overhead.

Overall, the findings support a cautious but constructive interpretation of quantum computing for optimization. Quantum algorithms introduce computational mechanisms that are genuinely different from classical search, including amplitude amplification, interference, Hamiltonian evolution, and constrained quantum-state exploration. These mechanisms create plausible opportunities for improving particular optimization tasks, but the available evidence does not support the broader conclusion that quantum computers have already achieved general practical superiority in combinatorial optimization. QAOA remains limited by depth, trainability, sampling, and noise; quantum annealing is affected by embedding and hardware restrictions; and minimum-finding algorithms require fault-tolerant circuits and efficient oracle construction. The strongest near-term direction is therefore likely to involve hybrid and problem-specific methods rather than fully quantum replacement of classical optimization. Classical computation can perform preprocessing, decomposition, parameter selection, and solution refinement, while the quantum component can be assigned carefully selected subproblems for which its physical dynamics may provide useful search behavior. Future studies should prioritize problem classes with structural properties that match quantum operations, improve constraint-preserving encodings, reduce circuit and measurement costs, and evaluate performance against the best available classical alternatives. Research should also separate three distinct claims: theoretical quantum speedup, experimental improvement on restricted instances, and end-to-end practical advantage. Conflating these claims creates unjustified expectations and weakens scientific interpretation. Quantum optimization remains a promising and intellectually significant field, but meaningful progress will depend less on increasing the number of algorithmic proposals than on producing rigorous, reproducible, hardware-aware, and application-relevant evidence.

CONCLUSION

This study concludes that quantum computing offers substantial theoretical and methodological potential for solving optimization problems, particularly through algorithms such as the Quantum Approximate Optimization Algorithm, quantum annealing, adiabatic quantum optimization, Grover-based minimum finding, and hybrid quantum–classical methods. These approaches introduce distinctive computational mechanisms, including superposition, interference, Hamiltonian evolution, amplitude amplification, and variational parameter optimization. Nevertheless, the reviewed evidence indicates that their effectiveness is highly dependent on problem structure, encoding strategy, circuit depth, hardware quality, classical optimization procedures, and the strength of the comparative baseline. Quantum algorithms cannot therefore be regarded as universally superior to classical optimization methods.

The analysis further shows that the principal barriers to practical quantum advantage include limited qubit numbers, hardware noise, restricted connectivity, high measurement costs, difficult parameter training, inefficient problem encoding, and substantial preprocessing and post-processing requirements. QAOA remains promising for gate-based combinatorial optimization but is constrained by trainability and circuit-depth limitations. Quantum annealing is useful for QUBO-compatible problems but is affected by embedding overhead and hardware-specific restrictions. Grover-based and minimum-finding algorithms provide clearer theoretical speedups, although their practical implementation requires fault-tolerant quantum computers and efficient oracle construction. The most realistic near-term direction is therefore the development of hybrid systems in which quantum processors handle carefully selected optimization subproblems while classical computers perform decomposition, parameter tuning, error mitigation, and solution refinement.

This study contributes a critical framework for distinguishing theoretical quantum speedup, small-scale experimental feasibility, and end-to-end practical advantage. The findings

emphasize that future research should prioritize transparent benchmarking, reproducible experiments, stronger classical comparisons, hardware-aware algorithm design, and efficient constraint-preserving formulations. Further studies should also examine larger and more diverse problem instances, investigate scalable parameter-initialization strategies, and assess total computational cost rather than quantum execution time alone. The principal limitation of this study lies in its qualitative literature-based design, which does not provide direct experimental validation or statistical meta-analysis. Future research should therefore combine systematic review with simulation and hardware-based experimentation to determine more precisely the conditions under which quantum optimization algorithms can deliver measurable scientific and industrial value.

DECLARATION OF AI AND AI ASSISTED TECHNOLOGIES IN THE WRITING PROCESS

During the preparation of this work, the authors used ChatGPT, developed by OpenAI, to assist with language refinement, grammatical correction, structural organization, and improvement of the academic style of the manuscript. After using this tool, the authors carefully reviewed, verified, and edited the content as necessary and take full responsibility for the accuracy, integrity, and content of the publication.

ACKNOWLEDGMENTS

I would like to thank you funding for this research. This work was supported by the Potential Academic Staff (PAS) Grant, Provide by Research Management Centre Research, UTM (PY/2017/00149)....

AUTHOR CONTRIBUTIONS

Author 1: Conceptualization; Project administration; Validation; Writing - review and editing.
Author 2: Conceptualization; Data curation; In-vestigation.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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