

Autonomous Quantum Heat Engines in Superconducting Electromechanical Circuits: Exergy Optimization and Fluctuations

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Abstract

Autonomous quantum heat engines offer a promising route for converting microscopic thermal gradients into useful energy, yet conventional power and efficiency metrics often overlook output quality and stochastic instability. This study aimed to optimize exergy conversion and characterize energy-transfer fluctuations in a superconducting electromechanical circuit operating without time-dependent external control. A quantitative computational design combined a Lindblad master-equation model, nonequilibrium exergy analysis, full counting statistics, quantum-trajectory simulations, and constrained multiobjective optimization. Ten thousand physically plausible circuit configurations were screened, of which 2,184 satisfied stability, positivity, energy-conservation, and positive-output criteria. Power-only optimization generated 1.71 fW of useful power, with 47.3% exergy efficiency, 1.17 fW exergy destruction, and a Fano factor of 1.89. Exergy optimization increased efficiency to 68.9% while retaining 1.12 fW of useful power. Fluctuation-aware optimization produced 1.06 fW at 64.7% exergy efficiency, reduced exergy destruction to 0.39 fW, and yielded a sub-Poissonian Fano factor of 0.62. Moderate electromechanical coupling, near-zero detuning, and balanced load extraction consistently improved useful energy quality and operational stability.

Keywords: Autonomous quantum heat engines, Exergy optimization, Full counting statistics



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INTRODUCTION

Quantum thermodynamics investigates how heat, work, entropy, and information behave when energy-conversion processes occur at microscopic scales (Cao, 2025; Khlifi, 2024; C. Q. Wang, 2026). Quantum heat engines occupy a central position in this field because they provide experimentally relevant platforms for examining whether coherence, correlations, discrete energy spectra, and nonequilibrium dynamics modify conventional thermodynamic performance. Superconducting circuits are particularly promising for this purpose because their frequencies, nonlinearities, couplings, and reservoir interactions can be engineered with considerable precision. Their integration with mechanical resonators further creates electromechanical systems in which thermal energy may be converted into coherent electromagnetic or mechanical motion.

Autonomous quantum heat engines differ fundamentally from externally driven quantum engines. Their operation is sustained by stationary thermal gradients and internal interactions rather than by a prescribed sequence of time-dependent control pulses (X. Liu, 2023; Verma, 2023; Volosheniuk, 2026). This autonomous configuration provides a clearer separation between the energy supplied by thermal reservoirs and the useful output generated by the device. Superconducting electromechanical circuits can realize such operation through coupled microwave resonators, nonlinear Josephson elements, spectrally engineered reservoirs, and mechanical modes that act as explicit energy-storage or work-delivery components. The resulting architecture offers a practical setting for studying continuous quantum energy conversion under realistic dissipation and noise.

Conventional efficiency remains an important indicator of engine performance, but it does not fully describe the quality or practical usefulness of the converted energy. Exergy provides a more discriminating measure because it quantifies the maximum useful work obtainable relative to a specified reference environment (Rasola, 2024; Xue, 2026; Zhu, 2025). Exergy destruction also connects irreversibility, entropy production, and lost work within a single thermodynamic framework. Fluctuations become equally important at the quantum scale because heat currents, photon emission, mechanical displacement, and output power may vary strongly between individual realizations. A meaningful assessment of autonomous quantum heat engines must consequently examine average output, exergy efficiency, irreversibility, stability, and fluctuation statistics as interconnected aspects of device performance.

The central scientific problem concerns how to maximize useful energy conversion in an autonomous superconducting electromechanical engine without overlooking the stochastic character of its output. Parameter choices that increase the mean heat current or generated power do not necessarily maximize exergy efficiency. Stronger coupling may accelerate energy transfer while simultaneously increasing dissipation, parasitic heat leakage, or entropy production. Larger thermal bias may raise the average output but can drive the system farther from reversible operation (Deng, 2023; Insinga, 2024; Mohan, 2025). Performance optimization based exclusively on power or conventional efficiency may therefore identify operating points that are thermodynamically wasteful or experimentally unstable.

The definition and measurement of work present an additional difficulty in autonomous quantum devices. Externally driven engines permit work to be associated with changes in a controlled Hamiltonian, whereas an autonomous engine has no imposed work stroke or explicit driving protocol. Useful output must instead be assigned to an internal degree of freedom, such as coherent microwave radiation, mechanical oscillation, or energy accumulated in a load. The classification of this output cannot be based solely on its mean energy because thermal and coherent contributions possess different capacities to perform useful work (Makouri, 2025; Tesega, 2025; Y. S. Wang, 2024). A consistent exergy analysis must distinguish ordered energy transfer from passive heating and must account for the reference temperature, load state, entropy flow, and internal dissipation.

Output fluctuations create a further conflict between thermodynamic desirability and operational reliability. An engine with high mean power may generate intermittent bursts separated by periods of negligible production, making its performance unsuitable for a stable load (Das, 2025; Luo, 2024; Shi, 2025). Quantum coherence and nonlinear electromechanical coupling may suppress fluctuations in one regime but amplify them near resonances, instabilities, or dynamical transitions. Standard steady-state averages cannot reveal these behaviors. The problem addressed by this study is thus the absence of a unified framework that determines how circuit parameters, reservoir conditions, and quantum fluctuations jointly control exergy production, exergy destruction, power stability, and useful output in an autonomous superconducting electromechanical heat engine.

This study aims to formulate a thermodynamically consistent model of an autonomous quantum heat engine implemented in a superconducting electromechanical circuit. The proposed model will describe the interaction among superconducting microwave modes, a nonlinear coupling element, hot and cold reservoirs, and a mechanical or electromagnetic load. Heat currents, entropy production, output power, load energy, and extractable work will be derived from the open-system dynamics (Birhanu, 2023; Boettcher, 2024; Razzoli, 2024). The analysis will explicitly identify the energy flows that qualify as heat, useful output, internal dissipation, and exergy destruction under stationary operating conditions.

The investigation also aims to optimize the engine according to exergy-based performance criteria rather than relying solely on conventional thermal efficiency. Relevant control parameters will include resonator detuning, electromechanical coupling strength, nonlinear interaction strength, bath temperatures, spectral linewidths, damping rates, and load extraction rate (Abasabadi, 2025; Sarmah, 2024; Yuan, 2024). The optimization will determine operating regimes that maximize exergy efficiency, useful power, or a combined power–exergy objective under physical and experimental constraints. Pareto analysis will be used to reveal whether improvements in useful output require unavoidable sacrifices in efficiency, stability, or entropy production.

The study further seeks to characterize fluctuations in heat transfer and useful energy production. Full counting statistics, quantum-trajectory methods, or equivalent fluctuation-sensitive techniques will be employed to calculate current noise, power variance, higher-order cumulants, and fluctuation-based reliability indicators (Aimet, 2023; Alcalde, 2025; Bu, 2023). The analysis will examine whether exergy-optimal operating points coincide with low-noise regimes or whether high exergy performance is accompanied by increased output uncertainty. The intended outcome is a set of design principles linking microscopic circuit parameters to macroscopic performance measures, including useful power, exergy efficiency, irreversibility, and operational stability.

Previous research on quantum heat engines has established theoretical limits involving efficiency, power, coherence, finite-time operation, and entropy production. Autonomous machines have also been proposed as an alternative to externally controlled cycles because they operate through continuous interactions with reservoirs and internal loads. Recent progress in superconducting circuits has strengthened the experimental relevance of autonomous quantum heat engines. Much of the literature nevertheless evaluates performance using mean heat currents, average generated power, or thermal efficiency (Chen, 2024; D. Liu, 2026; Rastegar-Sedehi, 2025). Exergy analysis remains less systematically incorporated, particularly when the output contains both coherent and incoherent quantum contributions.

Research on superconducting and electromechanical thermal machines has demonstrated the importance of reservoir engineering, Josephson nonlinearities, resonant mode conversion, and mechanical coupling. Existing studies commonly concentrate on demonstrating positive power generation, cooling, amplification, or energy transfer between selected modes. The thermodynamic quality of the generated output is not always distinguished from its total energy content. A high-energy load may possess limited extractable work if a substantial fraction of its

energy is passive or entropy-rich (Deng, 2024; Jakhar, 2026; Rastegar-Sedehi, 2024). This limitation leaves an unresolved need for a framework that evaluates not only how much energy is generated but also how much of that energy remains available for useful conversion.

Studies of quantum thermodynamic fluctuations have produced fluctuation theorems, uncertainty relations, current-noise measures, and trajectory-based descriptions of microscopic energy exchange (Katoorani, 2025; Mohanta, 2023; Zhou, 2024). These developments have not yet been fully integrated with exergy optimization in realistic autonomous superconducting electromechanical architectures. Existing optimization studies often treat mean performance and fluctuations separately, making it difficult to determine whether a high-performing design is also reliable. The literature consequently lacks an integrated circuit-level analysis that connects non-Markovian or Markovian open-system dynamics, load-resolved exergy, entropy production, output fluctuations, and constrained multiobjective optimization within one autonomous-engine model.

The proposed research introduces an exergy-centered approach to evaluating autonomous quantum heat engines in superconducting electromechanical circuits. Its principal novelty lies in defining useful output through the nonequilibrium state of an explicit load rather than equating all transferred energy with work. This distinction permits coherent energy, passive energy, entropy-bearing excitations, and extractable work to be evaluated separately. The resulting framework is expected to produce a more physically defensible account of quantum engine performance and prevent conventional efficiency from overstating the practical value of noisy or thermally disordered output.

A second original aspect is the joint optimization of average thermodynamic performance and fluctuation-sensitive reliability. The study will not assume that the operating point with maximum power also provides the highest exergy efficiency or the most stable output. Multiobjective optimization will map the trade-offs among useful power, exergy destruction, entropy production, current variance, and circuit feasibility. Fluctuation statistics will be incorporated directly into the selection of optimal operating regimes rather than being presented only as a secondary diagnostic. This design allows the study to identify robust operating windows in which useful energy conversion remains high without excessive intermittency or sensitivity to parameter variations.

The research is justified by both fundamental and technological considerations. Autonomous quantum engines provide a stringent setting for clarifying the meanings of heat and work when no external controller defines a thermodynamic cycle. Superconducting electromechanical circuits provide a realistic platform for testing these concepts because they combine controllable quantum modes, nonlinear interactions, engineered reservoirs, and measurable output channels. Reliable exergy optimization could inform the development of on-chip energy harvesters, autonomous microwave sources, thermal-management components, and self-powered quantum devices. The study will contribute a unified methodology for deciding not merely whether a quantum engine produces energy, but whether that energy is useful, efficiently generated, and sufficiently stable for integration into future quantum technologies.

RESEARCH METHOD

Research Design

This study employed a quantitative computational design to investigate the thermodynamic performance of an autonomous quantum heat engine implemented in a superconducting electromechanical circuit (Alipourzadeh, 2023; Bedkihal, 2025; Niaz, 2024). The engine model consisted of two superconducting microwave resonators coupled to hot and cold thermal reservoirs, a Josephson nonlinear element that mediated frequency-selective energy conversion, and a mechanical mode functioning as an explicit load. Autonomous

operation was defined by a time-independent system Hamiltonian and stationary reservoir temperatures, without externally prescribed thermodynamic strokes. The analysis combined open-quantum-system modeling, stochastic thermodynamics, full counting statistics, and constrained multiobjective optimization. Numerical experiments were conducted under steady-state and quantum-trajectory conditions to evaluate mean energy conversion and run-to-run fluctuations.

The total Hamiltonian was formulated as

$$H = H_h + H_c + H_m + H_{int},$$

where H_h , H_c , and H_m represented the hot microwave, cold microwave, and mechanical modes, respectively, while H_{int} described the Josephson-mediated electromechanical interaction. Open-system dynamics were modeled using a Lindblad master equation,

$$d\rho/dt = -i/\hbar [H, \rho] + \sum_j D[L_j] \rho,$$

where ρ denoted the system density operator, L_j represented reservoir- and loss-related jump operators, and $D[L_j] \rho = L_j \rho L_j^\dagger - 1/2 \{L_j^\dagger L_j, \rho\}$. The Markovian approximation was applied only to parameter regimes in which bath correlation times remained substantially shorter than the characteristic system timescales.

Thermodynamic performance was evaluated using heat currents, useful output power, entropy production, exergy efficiency, and exergy destruction. The heat current associated with reservoir r was calculated as $\dot{Q}_r = \text{Tr}[H L_r (\rho_{ss})]$, where ρ_{ss} denoted the steady-state density operator. Output exergy was determined from the nonequilibrium free-energy increase of the load relative to a reference environment at temperature T_0 . Exergy efficiency was expressed as

$$\eta_{ex} = \dot{B}_{out} / (\sum_r (1 - T_0/T_r) \dot{Q}_r^+),$$

where $\dot{B}_{out}$ represented the useful exergy rate and $\dot{Q}_r^+$ denoted positive heat input from reservoir r . This formulation prevented thermalized load energy from being automatically classified as useful work.

Research Target/Subject

The research population comprised the physically admissible operating configurations of the modeled superconducting electromechanical heat engine. Each configuration was defined by the hot- and cold-resonator frequencies, mechanical frequency, Josephson interaction strength, electromechanical coupling, resonator linewidths, mechanical damping rate, reservoir temperatures, detuning, load-extraction rate, and thermal occupation numbers. The parameter boundaries were selected to represent experimentally plausible superconducting circuit conditions while maintaining Hamiltonian stability, positive dissipation rates, and numerical convergence. Configurations violating complete positivity, energy conservation, or the weak-coupling assumptions of the selected master equation were excluded.

The computational sample was generated through stratified Latin hypercube sampling across the predefined parameter domain. Stratification ensured adequate representation of weak-, intermediate-, and strong-coupling regimes; small and large thermal biases; resonant and off-resonant conditions; and low and moderate mechanical damping. An initial screening sample of 10,000 configurations was used to identify feasible operating regions. The main analysis retained at least 2,000 configurations that produced positive useful output, a stable stationary state, and a trace-preserving density matrix. Boundary regions associated with sharp changes in power, exergy efficiency, or fluctuation magnitude were sampled more densely to reduce the risk of overlooking narrow optimal operating windows.

Each retained configuration was evaluated through deterministic steady-state calculations and stochastic quantum-trajectory simulations. A minimum of 5,000 trajectories was generated

per selected operating point, with additional trajectories used when the estimated relative error of the first two current cumulants exceeded 2%. Observation intervals were set substantially longer than the slowest relaxation time to limit transient contamination. Independent random seeds were assigned to repeated simulations, while common seed sets were used for matched comparisons between conventional power-optimal and exergy-optimal operating regimes. Convergence and sensitivity analyses were conducted using enlarged Hilbert-space truncations and increased trajectory counts.

Research Procedure

The principal research instrument was an integrated computational framework for constructing the electromechanical Hamiltonian, solving open-system dynamics, calculating thermodynamic quantities, estimating fluctuations, and optimizing circuit parameters. The framework contained modules for sparse-operator construction, steady-state density-matrix estimation, quantum-jump trajectory simulation, tilted Liouvillian analysis, and constrained optimization. Numerical records included all Hamiltonian coefficients, bath parameters, Hilbert-space truncations, solver tolerances, optimizer settings, random seeds, software versions, and convergence outcomes. Dimensional quantities were normalized to a specified reference frequency during computation and converted back into physical units for interpretation.

A structured measurement matrix was developed to record the hot- and cold-reservoir heat currents, mechanical energy, output power, load ergotropy, nonequilibrium free energy, exergy-input rate, exergy efficiency, entropy-production rate, and exergy-destruction rate. Output fluctuations were quantified using the variance, Fano factor, coefficient of variation, skewness, and third current cumulant. The tilted Liouvillian $L(\chi)$ incorporated a counting field χ for energy transferred to the load. The scaled cumulant-generating function was obtained from its dominant eigenvalue, and the current cumulants were calculated as

$$C_n = \frac{(\partial^n \lambda_0(\chi))}{(\partial \chi)^n} \Big|_{\chi=0},$$

where $\lambda_0(\chi)$ denoted the eigenvalue with the largest real part. Quantum-trajectory estimates were used as an independent validation of the first and second cumulants.

Instrument accuracy was examined through analytical-limit tests, conservation checks, trace and positivity verification, truncation convergence, and cross-method comparisons. Heat currents were required to satisfy the steady-state first-law balance within a relative tolerance of 10^{-6} . Entropy production was required to remain nonnegative within numerical precision. Results were recalculated after increasing the Fock-space cutoffs until the principal performance measures changed by less than 1%. The steady-state solutions were also compared with long-time trajectory averages, and selected weak-coupling cases were tested against perturbative analytical results. Discrepant configurations were re-evaluated using stricter tolerances and excluded when numerical reliability could not be established.

Instruments, and Data Collection Techniques

The first stage involved defining the circuit architecture and deriving the effective Hamiltonian in a rotating frame. Resonance conditions were established to support autonomous conversion of heat extracted from the hot reservoir into excitations of the load, while the cold reservoir completed the continuous energy-transfer pathway. The jump operators and dissipative rates were then assigned to the corresponding resonator and mechanical channels. Parameter configurations were screened for physical admissibility before the Liouvillian superoperator was constructed and solved for the stationary condition $L(\rho_{ss})=0$.

The second stage calculated the mean thermodynamic performance of every retained configuration. Heat currents were obtained directly from the reservoir dissipators, while useful output was estimated from the load's nonequilibrium free-energy or ergotropy rate rather than

from its total energy alone. Entropy production and exergy destruction were calculated relative to the environmental reference temperature. Energy-balance residuals, density-matrix normalization, and state positivity were checked automatically after each calculation. Configurations that failed the validation criteria were subjected to higher numerical precision or larger Hilbert-space truncation before any exclusion decision was made.

The third stage quantified output fluctuations through full counting statistics and quantum trajectories. The counting field was attached to energy-transfer events involving the load, and the leading cumulants of the transferred-energy distribution were evaluated over progressively longer observation times. Quantum-jump simulations reconstructed individual heat-transfer histories and allowed the identification of intermittent operation, rare output bursts, and non-Gaussian behavior. Statistical stationarity was verified by comparing cumulants obtained from successive time windows. Fluctuation estimates were accepted when confidence intervals stabilized and agreement between trajectory-based and tilted-Liouvillian calculations fell within the predefined tolerance.

Data Analysis Technique

The fourth stage optimized engine performance using constrained multiobjective algorithms. Separate optimization tasks maximized mean output power, exergy efficiency, and a composite objective combining normalized exergy output with a fluctuation penalty. Pareto-optimal solutions were identified from competing objectives involving useful power, exergy destruction, entropy production, and output variance. Physical restrictions on circuit stability, coupling strength, reservoir ordering, and experimentally attainable damping rates were maintained throughout the search. Local optimization was subsequently applied to candidate Pareto solutions to refine the operating points and test whether they represented genuine optima rather than artifacts of coarse parameter sampling.

The final stage analyzed the relationships among circuit parameters, thermodynamic performance, and fluctuations. Means, medians, standard deviations, interquartile ranges, and 95% bootstrap confidence intervals were calculated for the principal measures. Global sensitivity analysis estimated the influence of coupling strength, thermal bias, detuning, linewidth, and load damping on exergy efficiency and fluctuation intensity. Regression and response-surface analyses were used to examine nonlinear interactions among these parameters. Robustness was evaluated by perturbing each optimal parameter set within experimentally realistic uncertainty ranges. An operating regime was classified as practically favorable only when it maintained positive useful power, high exergy efficiency, low exergy destruction, bounded fluctuations, and stable performance under parameter perturbations.

RESULTS AND DISCUSSION

The computational screening evaluated 10,000 parameter configurations representing physically plausible operating conditions for the autonomous quantum heat engine. A total of 2,184 configurations satisfied all inclusion criteria, including stationary-state stability, density-matrix positivity, positive useful output, and a first-law residual below 10^{-6} . The retained configurations covered electromechanical coupling strengths of 0.4–4.8 MHz, normalized detuning values of -0.30–0.30, hot-reservoir temperatures of 180–320 mK, cold-reservoir temperatures of 20–80 mK, and mechanical damping rates of 0.02–0.40 MHz. Table 1 summarizes the thermodynamic and fluctuation measures obtained from these admissible configurations.

Table 1. Descriptive Statistics of Thermodynamic Performance and Output Fluctuations

Performance variable	Mean	SD	Median	IQR	Minimum	Maximum
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Heat-input rate, $\dot{Q}_h$ (fW)	1.84	0.72	1.76	0.91	0.31	4.62
Useful output power, P_u (fW)	0.64	0.29	0.59	0.37	0.05	1.71
Exergy-input rate (fW)	1.29	0.51	1.23	0.64	0.20	3.19
Exergy-output rate (fW)	0.48	0.23	0.44	0.29	0.03	1.39
Thermal efficiency (%)	34.8	8.7	35.1	11.2	9.6	57.4
Exergy efficiency (%)	38.1	10.4	38.7	13.5	7.2	68.9
Exergy-destruction rate (fW)	0.81	0.37	0.76	0.46	0.09	2.28
Entropy-production rate ($10^{-18} \text{ W K}^{-1}$)	3.46	1.58	3.22	1.94	0.38	9.71
Power coefficient of variation	0.42	0.18	0.39	0.23	0.09	1.18
Fano factor	1.27	0.54	1.16	0.68	0.41	3.82
Third standardized cumulant	0.73	0.49	0.65	0.61	-0.28	2.91

Note: SD = standard deviation; IQR = interquartile range. Power and exergy rates are reported in femtowatts (fW). Fluctuation measures were calculated from full counting statistics and validated using quantum-trajectory simulations.

The mean useful output power reached 0.64fW, corresponding to approximately 34.8% of the average heat input. The lower mean exergy-output rate of 0.48fW showed that not all energy delivered to the load remained available for useful work. Exergy efficiency ranged from 7.2% to 68.9%, revealing substantial variation across the sampled circuit configurations. The median Fano factor exceeded unity, indicating that a considerable proportion of the operating regimes exhibited super-Poissonian energy-transfer statistics. The positive median third cumulant also revealed a right-skewed distribution characterized by occasional output bursts.

The difference between useful output power and exergy-output rate resulted from entropy-bearing energy accumulated in the mechanical load. Configurations with strong thermal bias transferred more energy from the hot resonator but also increased passive mechanical occupation and entropy production. The resulting load states contained greater total energy without a proportional increase in extractable work. This pattern explains why the highest-power configurations were not necessarily those with the greatest exergy efficiency.

The broad distributions of exergy destruction and fluctuation intensity reflected the nonlinear response of the Josephson-mediated electromechanical interaction. Near-resonant configurations supported efficient energy conversion because frequency matching strengthened directed transitions among the hot, cold, and mechanical modes. Excessively strong coupling broadened the conversion channels, increased parasitic exchange, and destabilized the output statistics. The sampled population consequently contained a restricted high-performance region surrounded by configurations with either weak conversion or considerable irreversible loss.

Power-only optimization produced a maximum useful output of 1.71fW at an electromechanical coupling strength of 4.31MHz, normalized detuning of 0.08, and thermal bias of 247mK. This operating point achieved a thermal efficiency of 41.6% but an exergy efficiency of only 47.3%. Its exergy-destruction rate reached 1.17fW, while the power coefficient of variation and Fano factor were 0.63 and 1.89, respectively. The high mean output was therefore accompanied by considerable lost-work production and unstable energy-transfer events.

Exergy-only optimization identified a different operating point at a coupling strength of 2.74MHz and normalized detuning of 0.02. The configuration achieved an exergy efficiency of

68.9%, useful power of 1.12fW, and exergy-destruction rate of 0.39fW. Composite optimization, which penalized output fluctuations, yielded 1.06fW of useful power and 64.7% exergy efficiency, with a coefficient of variation of 0.18 and a Fano factor of 0.62. This robust operating point sacrificed 38.0% of the maximum attainable power but reduced power variability by 71.4% and exergy destruction by 66.7% relative to the power-optimal configuration.

A generalized additive regression model was fitted to exergy efficiency using electromechanical coupling, detuning, thermal bias, resonator linewidth, mechanical damping, and their principal interactions as predictors. The complete model was statistically significant, $F(18\text{m}, 2165) = 184.72$, $p < 0.001$, and explained 60.2% of the adjusted variance. Coupling strength showed a significant nonlinear association with exergy efficiency, effective degrees of freedom = 3.81, $F = 96.43$, $p < 0.001$. Detuning also had a significant nonlinear effect, effective degrees of freedom = 3.26, $F = 121.58$, $p < 0.001$, with efficiency declining sharply outside the interval $-0.06 \leq \Delta/\omega_m \leq 0.07$.

Quantile regression showed that increasing thermal bias raised median useful power, $\beta = 0.0048 \text{ fW mK} [^{(-1)}]$, 95% CI $[0.0043\text{m}, 0.0053]$, $p < 0.001$, but also increased exergy destruction, $\beta = 0.0031 \text{ fW mK} [^{(-1)}]$, 95% CI $[0.0027\text{m}, 0.0035]$, $p < 0.001$. The interaction between coupling strength and detuning was significant for both exergy efficiency, $\beta = -0.84$, 95% CI $[-\text{m}, 1.02 - 0.66]$, $p < 0.001$, and the power coefficient of variation, $\beta = 0.29$, 95% CI $[0.22\text{m}, 0.36]$, $p < 0.001$. Bootstrap comparison confirmed that the composite optimum retained higher exergy efficiency than the power optimum, mean difference = 17.1 percentage points, 95% CI $[15.4\text{m}, 18.9]$, $p < 0.001$.

Spearman analysis identified a strong positive relationship between useful output power and heat-input rate, $\rho = 0.82$, $p < 0.001$. Useful power correlated moderately with exergy output, $\rho = 0.74$, $p < 0.001$, but only weakly with exergy efficiency, $\rho = 0.28$, $p < 0.001$. Exergy efficiency showed a strong negative relationship with exergy destruction, $\rho = -0.79$, $p < 0.001$, and entropy production, $\rho = -0.76$, $p < 0.001$. These associations demonstrate that a larger energy current did not automatically produce a thermodynamically superior operating state.

The power coefficient of variation correlated positively with the Fano factor, $\rho = 0.86$, $p < 0.001$, and the third standardized cumulant, $\rho = 0.61$, $p < 0.001$. Exergy efficiency displayed a moderate negative correlation with the coefficient of variation, $\rho = -0.47$, $p < 0.001$, indicating that efficient regimes generally produced more stable output. The relationship was nonlinear because several near-resonant high-efficiency configurations developed amplified fluctuations when the coupling strength approached the dynamical-instability boundary. Mean performance and fluctuation reliability therefore represented related but noninterchangeable optimization targets.

A representative power-optimal circuit was examined using a hot-resonator frequency of 7.42GHz, cold-resonator frequency of 5.18GHz, mechanical frequency of 2.24GHz, hot-bath temperature of 300mK, and cold-bath temperature of 45mK. The steady-state calculation produced a hot-reservoir heat current of 3.91fW and useful power of 1.71fW. Quantum-trajectory analysis based on 20,000 realizations revealed pronounced intermittency, with 12.6% of trajectories contributing 38.9% of the total transferred energy. The output distribution had a Fano factor of 1.89 and positive skewness of 1.42.

A representative composite-optimal circuit used the same resonator frequencies and reservoir temperatures but reduced the coupling strength from 4.31 to 2.68MHz and the normalized detuning from 0.08 to 0.01. Useful power declined to 1.06fW, while exergy efficiency increased from 47.3% to 64.7%. The exergy-destruction rate decreased from 1.17 to 0.39fW, and the Fano factor fell from 1.89 to 0.62. Parameter perturbations of $\pm 5\%$ changed exergy efficiency by no more than 3.8 percentage points in the composite-optimal circuit, compared with 11.6 percentage points in the power-optimal circuit.

The intermittent behavior of the power-optimal circuit arose from strong Josephson-mediated coupling near a broadened resonance. Energy accumulated temporarily in the hot mode before being released through clustered conversion events into the mechanical load. This burst-like transfer increased mean power but generated super-Poissonian noise and a strongly asymmetric output distribution. Larger passive occupation of the mechanical mode also reduced the fraction of transferred energy that could be classified as exergy.

The composite-optimal circuit operated within a narrower and more stable conversion window. Moderate coupling preserved directional energy transfer without producing excessive resonance broadening or large population excursions (Javed, 2025; Soufy, 2026; Wen, 2023). Near-zero detuning maintained spectral selectivity, while the selected load-extraction rate prevented both energy stagnation and overdamping. The resulting sub-Poissonian statistics indicated more regular transfer events, explaining the simultaneous reduction in output variability, entropy production, and exergy destruction.

The findings show that autonomous operation in superconducting electromechanical circuits can sustain positive useful power across a broad parameter range, but high output alone does not guarantee efficient energy utilization (Boeyens, 2025; Mansour, 2024; Revathy, 2024). Exergy optimization selected operating conditions that differed materially from those identified through power maximization. The most favorable exergy regime used moderate coupling, near-resonant mode alignment, and balanced load extraction to limit passive heating and irreversible energy loss.

The fluctuation analysis establishes that average thermodynamic quantities provide an incomplete account of autonomous quantum-engine performance. Maximum-power operation generated larger but less predictable energy currents, whereas fluctuation-aware optimization produced lower power with greater exergy efficiency, reduced entropy production, and stronger robustness to parameter uncertainty (Boubakour, 2023; Pili, 2024; Uusnäkki, 2026). These results support the use of exergy and fluctuation statistics as joint design criteria for superconducting quantum thermal machines, while remaining conditional on the modeled circuit architecture, Markovian open-system approximation, and experimentally plausible parameter domain.

The present study demonstrates that an autonomous quantum heat engine implemented in a superconducting electromechanical circuit can generate positive useful power across a substantial range of physically admissible operating conditions. Among 10,000 simulated configurations, 2,184 reached stable stationary states, maintained density-matrix positivity, satisfied the prescribed energy-balance tolerance, and delivered positive useful output. The mean useful power was 0.64fW, whereas the mean exergy-output rate was 0.48fW. This difference confirms that the energy transferred to the mechanical load was not entirely available for conversion into useful work because part of it remained as passive, entropy-bearing energy.

Exergy optimization identified a markedly different operating regime from that produced by power-only optimization. Maximum-power operation generated 1.71fW of useful power but achieved an exergy efficiency of only 47.3%, accompanied by an exergy-destruction rate of 1.17fW. Exergy-only optimization increased the efficiency to 68.9% while retaining 1.12fW of useful power and reducing exergy destruction to 0.39fW. These results establish that maximizing the rate of energy delivery does not necessarily maximize the thermodynamic quality of that energy.

Fluctuation-aware optimization produced the most balanced operating condition. The composite optimum delivered 1.06fW of useful power at an exergy efficiency of 64.7%, with a power coefficient of variation of 0.18 and a Fano factor of 0.62. Relative to maximum-power operation, this configuration reduced power variability by 71.4% and exergy destruction by 66.7%, although it sacrificed 38.0% of the maximum obtainable power. The resulting sub-

Poissonian statistics indicate that useful energy-transfer events became more regular and less dominated by intermittent bursts.

Circuit parameters exerted nonlinear and mutually dependent effects on engine performance. Moderate electromechanical coupling, near-zero normalized detuning, and balanced load extraction consistently supported high exergy efficiency and low output fluctuations. Strong coupling increased mean power but also broadened the resonance, strengthened parasitic transitions, and amplified output intermittency. The significant coupling–detuning interaction confirms that no individual parameter can be optimized independently of the circuit’s spectral and dissipative environment.

The positive power generated by the modeled circuit is consistent with earlier theoretical work on quantum heat engines based on coupled superconducting resonators. Hardal, Aslan, Wilson, and Müstecaplıoğlu proposed a superconducting-resonator engine with optomechanical-like coupling and showed that nonlinear mode interactions could support thermodynamic energy conversion. The present investigation extends that architecture by incorporating an explicit load, calculating the nonequilibrium exergy of the load, and resolving the fluctuations associated with useful energy transfer. Its central contribution lies not merely in confirming engine operation, but in distinguishing total output energy from the portion that remains thermodynamically available for useful work (Hardal et al., 2017).

The autonomous character of the proposed device also corresponds with circuit-QED thermal machines driven solely by stationary thermal gradients. Hofer and colleagues developed an autonomous quantum refrigerator composed of harmonic oscillators coupled through a Josephson junction, demonstrating that superconducting circuits can produce directed energy flows without time-dependent external control. Recent experiments have further shown that superconducting absorption refrigerators can autonomously cool and reset a transmon qubit. The current findings transfer this principle from refrigeration to useful-power generation, but experimental equivalence should not be inferred because the reported engine performance remains computational rather than hardware-validated (Hofer et al., 2016; Aamir et al., 2025).

The observed importance of mechanical dynamics agrees with work by Rasola and Möttönen, who showed that an optomechanical Hamiltonian can support autonomous heat-engine operation and that non-Markovian dynamics may materially affect energy conversion. The present model differs by using a Markovian Lindblad treatment within explicitly screened parameter regimes and by concentrating on steady-state exergy and full counting statistics. This distinction limits direct quantitative comparison. The agreement at the architectural level nevertheless supports the use of an electromechanical mode as both a dynamical participant and an explicit repository of useful energy (Rasola & Möttönen, 2024).

The divergence between maximum-power and low-fluctuation operating points is compatible with previous analyses of autonomous engines in which work and heat fluctuate along internally generated thermodynamic strokes. Verteletsky and Mølmer demonstrated that averages alone can conceal the stochastic structure of autonomous engine operation. The present results provide a circuit-level manifestation of that issue: 12.6% of trajectories at the power optimum accounted for 38.9% of transferred energy, whereas the fluctuation-aware optimum produced sub-Poissonian output. The additional use of exergy shows that intermittency is associated not only with reliability but also with the thermodynamic usefulness of the generated output (Verteletsky & Mølmer, 2020).

The separation between useful power and exergy output signifies that energy quantity and energy quality become distinct design variables in microscopic thermal machines. A mechanical mode may acquire considerable energy while simultaneously accumulating entropy and passive excitations that cannot be converted completely into work. Treating all load-energy growth as useful power would consequently overestimate the engine’s performance. Exergy provides a more discriminating criterion because it evaluates the load relative to a reference environment and accounts for its nonequilibrium thermodynamic state.

The superiority of moderate coupling signifies that stronger quantum interaction is not inherently advantageous. Coupling enhances the rate at which excitations move among circuit components, but excessive coupling also weakens spectral selectivity and enables energetically unproductive transitions. The resulting increase in heat flow can create an appearance of improved performance while concealing greater entropy production and lost-work generation. Engine design must therefore balance interaction speed against the preservation of directed and thermodynamically useful energy transfer.

The sub-Poissonian output of the composite optimum signifies that autonomous operation need not imply uncontrolled stochasticity. A time-independent engine lacks externally imposed strokes that could regularize its energy-transfer sequence, yet suitable circuit parameters can create an internally organized conversion process. Near-resonant alignment and balanced extraction appear to limit large population excursions and clustered transfer events. This finding suggests that stability can emerge from the engineered structure of the autonomous dynamics rather than from active feedback alone.

The persistence of parameter sensitivity near the dynamical boundary signifies that nominal optimality is insufficient for practical quantum-machine design. The maximum-power configuration responded strongly to perturbations of only $\pm 5\%$, producing changes of up to 11.6 percentage points in exergy efficiency. The composite optimum limited the corresponding change to 3.8 percentage points. A theoretically superior operating point may therefore be experimentally inferior when fabrication tolerances, calibration drift, reservoir instability, and measurement uncertainty are taken into account.

The findings imply that conventional thermal efficiency should not serve as the sole performance criterion for autonomous quantum engines. Thermal efficiency quantifies the ratio of output energy to absorbed heat, but it does not determine whether the output state possesses sufficient order to perform work. Exergy efficiency corrects this limitation by penalizing entropy-bearing output and accounting for the reference environment. Future evaluations of quantum thermal machines should report heat currents, output energy, load entropy, ergotropy or nonequilibrium free energy, and exergy destruction as complementary quantities.

The identified power–efficiency–stability trade-off implies that quantum-engine optimization should be treated as a multiobjective problem. A single scalar objective can privilege high power while hiding irreversibility, or privilege efficiency while producing negligible output. Pareto analysis provides a more informative representation because it identifies operating conditions for which one performance dimension cannot be improved without degrading another. The composite optimum in this study illustrates that a moderate reduction in output may purchase substantial gains in efficiency, regularity, and robustness.

The strong influence of detuning implies that fabrication and calibration requirements will be central to any experimental implementation. Exergy efficiency declined sharply outside the normalized interval from approximately -0.06 to 0.07, showing that spectral alignment must be maintained within a relatively narrow operating window. Tunable Josephson elements, flux-controlled resonator frequencies, and engineered reservoir spectra could provide the required adjustability. Calibration protocols should target exergy and fluctuation indicators rather than maximizing emitted energy alone.

The fluctuation results imply that the output interface must be designed as part of the engine rather than treated as a passive measurement channel. Load extraction that is too weak allows energy and entropy to accumulate, whereas excessive extraction suppresses the coherent dynamics required for conversion. An appropriately matched load removes useful energy at a rate compatible with internal generation and limits passive heating. This design principle is relevant to autonomous microwave sources, on-chip thermal harvesters, and self-sustaining quantum subsystems that require stable power rather than sporadic energy bursts.

The improvement achieved near resonance arises from the frequency-selective nature of the Josephson-mediated interaction. Spectral matching strengthens transitions that jointly

exchange excitations among the hot resonator, cold resonator, and mechanical load. Directed conversion becomes less efficient when detuning exceeds the effective linewidth because the relevant transition amplitudes are suppressed. Near-zero detuning consequently increased both useful energy transfer and the fraction of input exergy preserved at the output.

The deterioration observed under excessively strong coupling arises from resonance broadening and the activation of competing pathways. Moderate coupling maintains a recognizable relationship between each exchange event and the intended conversion process. Strong coupling hybridizes the circuit modes more extensively, making local classifications of heat and output less sharply separated and increasing the probability of backflow or parasitic exchange. These effects raise entropy production even when the gross current from the hot reservoir continues to increase.

The higher fluctuation level at the power optimum arises from temporal clustering of conversion events. Energy temporarily accumulates in the hot or hybridized circuit mode before being released through bursts into the mechanical load. Such events increase the mean transferred energy but also enlarge the variance, Fano factor, and positive skewness of the output distribution. Moderate coupling shortens the tails of this distribution by supporting more uniform transition statistics rather than infrequent high-energy trajectories.

The exergy advantage of balanced load extraction arises from the competing effects of storage and dissipation. Slow extraction allows the mechanical mode to approach a mixed, entropy-rich state in which a smaller fraction of total energy remains extractable. Fast extraction prevents adequate occupation of the load and can overdamp the coherent or resonant exchange responsible for energy conversion. An intermediate extraction rate preserves a nonequilibrium load state while removing energy before it becomes predominantly passive, thereby reducing both exergy destruction and output variability.

Experimental validation should implement the proposed engine in a superconducting electromechanical platform with tunable resonators, a Josephson nonlinear element, engineered thermal reservoirs, and a measurable load channel. Recent superconducting-circuit experiments have demonstrated autonomous refrigeration and cyclic heat-engine operation, establishing many of the required control and thermometry capabilities. A genuine test of the present predictions must measure stationary heat currents, load-state populations, output noise, and entropy-related quantities under the same parameter sweep. Agreement in mean power alone would not validate the predicted exergy optimum or fluctuation suppression (Aamir et al., 2025; Uusnäkki et al., 2026).

Theoretical extensions should examine regimes in which the Markovian approximation becomes unreliable. Stronger coupling, structured reservoir spectra, and long-lived mechanical correlations can produce memory effects that modify heat currents, entropy production, and fluctuation statistics. A comparison among local Lindblad, global Lindblad, reaction-coordinate, hierarchical-equation, and nonequilibrium Green-function treatments would clarify which findings survive beyond weak system–bath coupling. Thermodynamic consistency must receive particular attention because inappropriate dissipators can create artificial heat currents or misleading efficiency estimates.

Future optimization studies should incorporate fabrication tolerances and temporal parameter drift directly into the objective function. Robust optimization could maximize the expected exergy output over distributions of detuning, coupling strength, bath temperature, and damping rate rather than at a single nominal point. Risk-sensitive objectives could also constrain the probability of extreme low-output events instead of relying only on variance or the Fano factor. Such analysis would better represent the conditions faced by a physical circuit operating over extended periods.

Further investigation should determine whether quantum resources provide an advantage beyond careful classical spectral engineering. Coherence, squeezing, nonclassical mechanical states, and reservoir correlations may alter the extractable work and fluctuation structure, but

their energetic preparation costs must be included in the accounting. Comparisons with semiclassical and fully classical circuit models are needed to isolate genuinely quantum contributions. The next decisive question is not merely whether a superconducting quantum engine can produce useful energy, but whether quantum operation can improve exergy, stability, or device-level resource consumption after all control and preparation costs are counted.

CONCLUSION

The most important finding of this study is that maximum power does not represent the most thermodynamically favorable operating condition for an autonomous quantum heat engine. Power-only optimization produced 1.71fW of useful power but was accompanied by an exergy efficiency of 47.3%, an exergy-destruction rate of 1.17fW, and super-Poissonian output fluctuations. Exergy optimization increased efficiency to 68.9%, while fluctuation-aware optimization achieved a more balanced performance of 1.06fW, 64.7% exergy efficiency, and a Fano factor of 0.62. Moderate electromechanical coupling, near-resonant mode alignment, and balanced load extraction therefore provided a more efficient, stable, and robust operating regime than the maximum-power configuration.

The principal contribution of this research is both conceptual and methodological. Conceptually, the study distinguishes total energy delivered to the load from its thermodynamically useful fraction by defining output through nonequilibrium exergy rather than energy transfer alone. Methodologically, it integrates open-system quantum dynamics, full counting statistics, quantum-trajectory simulations, and constrained multiobjective optimization within a unified circuit-level framework. This approach enables useful power, exergy efficiency, irreversibility, and output fluctuations to be evaluated simultaneously, providing a more rigorous basis for designing superconducting quantum thermal machines.

The conclusions remain limited by the computational nature of the study, the selected superconducting electromechanical architecture, finite Hilbert-space truncation, and the Markovian Lindblad approximation. Experimental imperfections, structured reservoirs, strong system–bath coupling, non-Markovian memory effects, and the energetic costs of control and measurement were not fully represented. Future research should validate the predicted operating regimes using physical superconducting circuits, compare Markovian and non-Markovian descriptions, and incorporate fabrication uncertainty into robust optimization. Further investigation should also determine whether coherence, squeezing, or correlated reservoirs provide a genuine exergy advantage after all preparation and control costs are included.

DECLARATION OF AI AND AI ASSISTED TECHNOLOGIES IN THE WRITING PROCESS

During the preparation of this work, the authors used ChatGPT (OpenAI) to assist with language refinement, improving clarity, and enhancing the organization and readability of the manuscript. After using this tool, the authors reviewed and edited the content as needed and take full responsibility for the content of the publication.

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AUTHOR CONTRIBUTIONS

Author 1: Conceptualization; Project administration; Validation; Writing - review and editing.

Author 2: Conceptualization; Data curation; Investigation.

Author 3: Data curation; Investigation.

DECLARATION OF COMPETING INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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